Quantum Pump for Fractional Charge

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PACS 73.23.-b – Electronic transport in mesoscopic systems
PACS 71.10.Pm – Fermions in reduced dimensions (anyons, composite fermions, Luttinger liquid, etc.)

Abstract. - We propose a theoretical scenario for pumping of fractionally charged quasi-particle in the context of $\nu = 1/3$ fractional quantum Hall liquid. We consider quasi-particle pumping across an anti-dot level tuned close to the resonance. Fractional charge pumping is achieved by slow and periodic modulation of coupling of the anti-dot level to left and right moving edges of a Hall bar set-up. This is attained by periodically modulating the gate voltages controlling the couplings. In order to obtain quantization of pumped charge in the unit of the electronic charge fraction (νe) per pumping cycle in the adiabatic limit, we argue that the only possibility is to tune the quasi-particle operator to be irrelevant from being relevant in the renormalization group sense, which can be accomplished by invoking quantum Hall line junctions into the Hall bar geometry. We also comment on possibility for experimental realization of the above scenario.

Introduction. – There has been a considerable interest in the direction of obtaining quantization of charge, pumped per cycle of pumping, in units of charge associated with the fundamental excitation of the quantum liquid in which the pump is operating. This issue was first addressed in a classic article by Thouless [1] followed by a series of theoretical [2–7] and experimental works [8–10]. Much of the research activity in this direction till date has been carried out in the context of Fermi liquids, which leads to quantization of pumped charge in units of the electronic charge e. Also in case of non-Fermi liquids, only pumping of electrons has been considered [11–15]. However the case of non-Fermi liquids with fractionally charged excitations has not been discussed much in the literature. An excellent candidate of direct relevance in this regard is the fractional quantum Hall liquid (FQHL). A very interesting scenario in this context was first proposed by Simon [16] where quantum charge pumping was considered across an anti-dot geometry in a two terminal Hall bar set-up for the case of $\nu = 1, 1/3$ and other abelian fractions of quantum Hall liquid. Charge pumping was achieved by periodic modulation of gate voltages which would push the left and the right moving edges of the Hall bar close to the anti-dot hence resulting in periodic modulation of tunnel-coupling of the left and right moving edges with the resonant level of the anti-dot. Here the left and right moving edges act as quasi-particle reservoirs for the quantum pump. As far as quantization of pumped charge in unit of νe in a single pumping cycle for $\nu = 1/m$ is concerned, this set-up has a serious short-coming. This is because, the quantization of pumped charge requires that the pumping contour in the parameter plane of couplings of the anti-dot to the left and right moving edges must enclose a quasi-particle resonance [4, 5, 7] such that the conductance on the pumping contour remains vanishingly small. This amounts to saying that, for obtaining quantization in the adiabatic limit the enclosed quasi-particle resonance should be very sharp. On the contrary, we must keep in mind that the quasi-particle tunnelling operator is a relevant operator [17] and hence in the limit of low temperature, zero bias and vanishingly small pumping frequency (IR limit) it will get renormalized and flow to the strong coupling limit destroying the quasi-particle resonance completely. In this letter, we propose a possible way to get around this hurdle. Our idea is to convert the quasi-particle operator from being relevant to an irrelevant (or marginal) one in the sense of renormalization group (RG) flow which will in turn result in sharpening of the quasi-particle resonance as we go to the IR limit. Theoretically, we show that this can be achieved by introducing strong, non-local inter-edge repulsive interactions within the left and right moving branches separately. To realize this within an experimentally feasible set-up, we propose to invoke two line junctions [18, 19] constructed

from the left and right moving branches, which operate as quasi-particle reservoirs for the quantum pump as shown in Fig. 1(b). Then, this line junction geometry naturally imbibes the possibility for realizing gate controlled inter-edge interactions leading to an irrelevant (in the sense of RG flow) quasi-particle tunnelling operator in the strong interaction limit.

In this letter we show that, it is possible to obtain an exact expression for pumped charge for a specific value of inter edge interaction of the line junction as the effective tunnelling from left to right edge through the anti-dot turns out to be marginal. Then, using weak interaction RG analysis [20], we find that the pumped charge gets quantized when the quasi-particle tunnelling operator is irrelevant and the quantization is destroyed, when tunnelling operator becomes relevant.

Proposed device and its theoretical modelling. – The proposed set-up comprises of an anti-dot, tunnel-coupled to two (half) line junctions acting as quasi-particle reservoirs (see Fig. 1(b)). We propose that the line junctions should be fabricated by *(i)* very thin etching of the 2DEG along the line perpendicular to the left and right moving edges from the two sides of the Hall bar such that the etching ends in the vicinity of the anti-dot region and then *(ii)* depositing top gates at the etched regions. The fine etching results in splitting of the FQHL in two parts with very closely spaced counter propagating edge states interacting only via Coulomb interaction. The etching of the 2DEG prevents tunnelling of electrons within the counter propagating edges of the line junction. In addition, by negatively biasing the top gates, V_{G1} and V_{G2}, the effective distance between the two counter propagating edges can be tuned resulting in tunability of strength of Coulomb interaction between the two edges. We also propose that the anti-dot should be produced by applying negative bias to, not just one, but two top gates (V_{G3} and V_{G4} in Fig. 1(b)) so that the tunnel coupling of the anti-dot with the line junction on its left and right side can be tuned independently.

We model the edge states forming the line junction using chiral Luttinger liquid theory [21]. We assume the inter-edge repulsive interaction between the counter propagating edges in each line junction is faithfully described by a screened Coulomb interaction as the long range part is expected to be screened by gates V_{G1} and V_{G2}, on top of the line junction. We also consider the anti-dot to be small enough so that the energy gap between two consecutive quasi-particle resonant levels, ΔE is larger than all other energy scales in the problem. Hence one can safely neglect the existence of all other levels in the anti-dot except for the one which is tuned to be the closest to the Fermi level in the lead and the anti-dot can be modelled as a single resonant level. In this limit, the quasi-particle creation and annihilation operators $(c, c^\dagger)$ represent a hard core anyon [22] as is clarified below. Even though in general the quasi-particle operator should obey exchange rules corresponding to the fractional statistics associated with the quasi-particle, for a single resonant level such statistical exchanges are irrelevant owing to the fact that there are no possibilities for exchange of quasi-particle in the set-up considered. Hence for all practical purposes, c and $c^\dagger$ can be treated as fermionic operators as their true statistics is irrelevant in the set-up considered and $\langle c^\dagger c\rangle$ can be either zero or one. The full Hamiltonian for our system can therefore be written as

$$\mathcal{H} = \mathcal{H}_R + \mathcal{H}_L + \mathcal{H}_{tunn} + \mathcal{H}_{ad}$$
$$= \sum_{i=R,L} \Bigg[\frac{\pi v}{\hbar\nu} \int_{-L/2}^{L/2} dx \left[(\rho_i(x))^2 + 2\lambda\rho_i(x)\rho_i(-x) \right] + \left[\Gamma_i(t) c^\dagger \psi_i(0) + h.c.\right] \Bigg] + \varepsilon(c^\dagger c)$$

Henceforth, we set $\hbar = 1$. Here, ε represents the energy of resonant level with respect to the Fermi-level in the leads and $\rho_{R/L}(x)$ represents the electronic density of the right (left) moving branch and can be expressed in terms of the bosonic field as $\rho_{R/L}(x) = \pm(1/2\pi)\partial_x\phi_{R/L}(x)$, which satisfies the commutation relation, $[\phi_i(x), \phi_i(x')] = \pm i\pi\nu\ \mathsf{Sgn}(x - x')$. c and $c^\dagger$ are the quasi-particle annihilation and creation operators in the anti-dot level which satisfy $\{c, c^\dagger\} = 1$ for the hard core anyon limit as discussed earlier. $\psi_{R/L}(x)$ is the quasi-particle annihilation operator in the line junction which is related to the bosonic fields via the standard bosonisation identity, $\psi_{R/L}(x) = (f_{R/L}/(2\pi\epsilon)^{\nu/2})e^{i\phi_{R/L}(x)}$ where ϵ is the short distance cut-off and $f_{R/L}$ is the Klein factor associated with the quasi-particle operator. Henceforth we shall drop the Klein factors as quasi-particle exchange processes are not relevant for transport through a single resonant level [23]. $\Gamma_{R/L}(t)$ is the time-dependent tunnel coupling and λ is the strength of screened Coulomb interaction between the counter propagating edges of the line junctions. We now define new fields

$$\varphi_{R/L} = \frac{\phi^+_{R/L} + \phi^-_{R/L}}{2}\ ; \quad \vartheta_{R/L} = \frac{\phi^+_{R/L} - \phi^-_{R/L}}{2}$$

where $\phi^\pm_{R/L} = \phi_{R/L}(\pm x)$ and the ϑ field satisfies the boundary condition, $\vartheta(x = 0) = 0$. These new fields diagonalize the line junction Hamiltonian to give

$$\mathcal{H}_{R/L} = \frac{v}{8\pi\nu} \int_0^{L/2} dx \left[k(\partial_x\varphi_{R/L})^2 + \frac{1}{k}(\partial_x\vartheta_{R/L})^2 \right]$$

where $v = v_0\sqrt{1-\lambda^2}$ is the renormalized velocity and $k = \sqrt{(1-\lambda)/(1+\lambda)}$ is the Luttinger liquid parameter for $\nu = 1$ case. Note that $0 \le k < 1$ for repulsive interaction ($\lambda > 0$), while for attractive interaction ($\lambda < 0$), $k > 1$. Now we re-scale the φ and ϑ fields as

$$\tilde\varphi_{R/L} = \sqrt{\frac{k}{\nu}}\,\varphi_{R/L}\ ; \quad \tilde\vartheta_{R/L} = \frac{1}{\sqrt{k\nu}}\,\vartheta_{R/L}$$

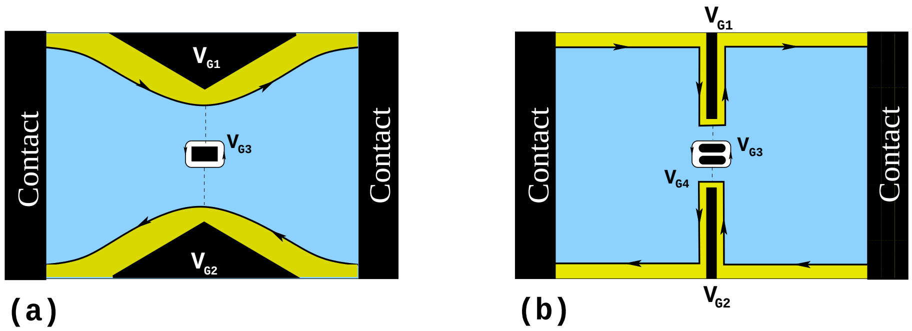

Fig. 1: (a) The conventional Hall bar geometry with an anti-dot in the center produced by a top gate, V_{G3}. (b) The proposed geometry with two half line junctions invoked with an anti-dot in the center which is produced by two top gates, V_{G3} and V_{G4}. The dotted line shows that there is tunnelling from the anti-dot to the left and right moving edges. The region containing quantum Hall liquid is shown in blue colour.

to obtain

$$\mathcal{H}_{R/L} = \frac{1}{8\pi}\int_0^{L/2} dx \left[(\partial_x \tilde{\varphi}_{R/L})^2 + (\partial_x \tilde{\vartheta}_{R/L})^2\right] \tag{1}$$

and

$$\mathcal{H}_{tunn} = \sum_{i=R,L} \frac{1}{(2\pi\epsilon)^{\nu/2k}} \Gamma_i(t)\, c^\dagger\, e^{i\sqrt{\nu/k}\,\tilde{\varphi}_i(0)} + h.c. \tag{2}$$

The fields, $\tilde{\vartheta}_{R/L}$ and $\tilde{\varphi}_{R/L}$ satisfy the commutation relations

$$\begin{aligned}[][\tilde{\vartheta}_{R/L}(x), \tilde{\vartheta}_{R/L}(x')] = [\tilde{\varphi}_{R/L}(x), \tilde{\varphi}_{R/L}(x')] = 0 \\ [\tilde{\varphi}_{R/L}(x), \tilde{\vartheta}_{R/L}(x')] = \pm i\frac{\pi}{2}\,\mathsf{Sgn}(x-x')\end{aligned} \tag{3}$$

When $\nu_{eff} = \nu/k$ is tuned to unity by tuning k, Eqs. (1)-(3) together with $\mathcal{H}_{ad}$ define a Hamiltonian which is identical to the bosonized version of the free fermionic Hamiltonian corresponding to the case of $\nu = 1$ quantum Hall state describing tunnelling of fermions from freely propagating left moving edge to a freely propagating right moving edge via an anti-dot level. The complete the mapping we define the new free chiral bosonic fields $\tilde{\phi}_R$ and $\tilde{\phi}_L$ corresponding to the $\nu_{eff} = 1$ theory which are related to $\tilde{\varphi}_{R/L}(x)$ and $\tilde{\vartheta}_{R/L}(x)$ by the following relation

$$\tilde{\varphi}_{R/L}(x) = \frac{\tilde{\phi}_{R/L}(x) + \tilde{\phi}_{R/L}(-x)}{2}$$
$$\tilde{\vartheta}_{R/L}(x) = \frac{\tilde{\phi}_{R/L}(x) - \tilde{\phi}_{R/L}(-x)}{2}$$

Such that $[\tilde{\phi}_{R/L}(x), \tilde{\phi}_{R/L}(x')] = \pm i\pi\mathsf{Sgn}(x-x')$. It is crucial to note that, in this new free fermionic theory, every time a fermion tunnels in or out of the right (left) edge into the anti-dot, it corresponds a net transfer of charge which is νe and not e. Once this mapping to the free fermion theory is established, it is straight-forward to calculate pumped charge in the adiabatic limit using the Brouwer's formula [24]. This is pursued in what follows.

Pumping for the case of $k = \nu$. – Given the mapping established in the previous section, we first write down the free fermionic Hamiltonian in terms of the Dirac fields as follows

$$\begin{aligned} H = -iv_F \int dx[\psi_R^\dagger \partial_x \psi_R - \psi_L^\dagger \partial_x \psi_L] + \varepsilon c^\dagger c + \\ [\Gamma_L(t) c^\dagger \psi_L(0) + \Gamma_R(t) c^\dagger \psi_R(0) + h.c.]\end{aligned}$$

Here the Fermi velocity, $v_F = v_0\sqrt{1-\lambda^2}$. Also the fields $\psi_{R/L}$ is related to the bosonic fields $\tilde{\phi}_{R/L}$ by the relation, $\psi_{R/L}(x) = 1/(2\pi\epsilon)^{1/2}\ e^{i\tilde{\phi}(x)_{R/L}}$. Here we have dropped Klein factors as they are not important for our case of single resonant level. We assume that $\Gamma_{L/R}(t)$ vary periodically with a period τ, which is much larger then all other time scales in the problem, so that we are in the adiabatic limit and hence we can work with the time-frozen Hamiltonian to find the instantaneous (adiabatic) eigenstates of the Hamiltonian. The Heisenberg equations of motion for the three fields ψ_R, ψ_L, c are

$$\begin{aligned} -i\partial_t \psi_R &= iv\partial_x \psi_R + \Gamma_R^\star c\delta(x) \\ -i\partial_t \psi_L &= -iv\partial_x \psi_L + \Gamma_L^\star c\delta(x) \\ i\partial_t c &= \varepsilon c + \Gamma_R \psi_R(0) + \Gamma_L \psi_L(0)\end{aligned} \tag{4}$$

Since for $x \neq 0$, The above equations (Eq. (4)) are just free particle equations of motion, hence we invoke plane wave solutions to obtain the scattering matrix as given below. For algebraic simplification, we define $c = \tilde{c}e^{-i\varepsilon t}$.

$$\psi_R(x,t) = \frac{1}{\sqrt{L}}\sum_\omega e^{i\omega(x/v-t)} \begin{cases} a_{R,\omega}, & x<0; \\ b_{R,\omega}, & x>0. \end{cases}$$
$$\psi_L(x,t) = \frac{1}{\sqrt{L}}\sum_\omega e^{i\omega(x/v+t)} \begin{cases} a_{L,\omega}, & x>0; \\ b_{L,\omega}, & x<0. \end{cases}$$
$$= \frac{1}{\sqrt{L}}\sum_\omega e^{i\omega(x/v-t)} \begin{cases} a_{L,-\omega}, & x<0; \\ b_{L,-\omega}, & x>0. \end{cases}$$
$$\tilde{c} = \sum_\omega e^{-i\omega t} c_\omega$$

Plugging these solutions into (4) we obtain

$$b_{R,\omega} = \frac{|\Gamma_L|^2 - |\Gamma_R|^2 - iv(\varepsilon-\omega)}{|\Gamma_R|^2 + |\Gamma_L|^2 - iv(\varepsilon-\omega)}\, a_{R,\omega} - \frac{2\Gamma_R^\star \Gamma_L}{|\Gamma_R|^2 + |\Gamma_L|^2 - iv(\varepsilon-\omega)}\, a_{L,-\omega}$$

$$b_{L,-\omega} = -\frac{2\Gamma_R \Gamma_L^\star}{|\Gamma_R|^2 + |\Gamma_L|^2 - iv(\varepsilon-\omega)}\, a_{R,\omega} + \frac{|\Gamma_R|^2 - |\Gamma_L|^2 - iv(\varepsilon-\omega)}{|\Gamma_R|^2 + |\Gamma_L|^2 - iv(\varepsilon-\omega)}\, a_{L,-\omega}$$

which gives us the scattering matrix

$$\begin{vmatrix} b_{R,\omega} \\ b_{L,-\omega} \end{vmatrix} = S \begin{vmatrix} a_{R,\omega} \\ a_{L,-\omega} \end{vmatrix}$$

Since we are only interested in the limit, $\omega \to 0$ of the problem, henceforth we shall only deal with S-matrix at $\omega = 0$. Following [3] we express the scattering matrix as

$$S = e^{i\theta} \begin{vmatrix} (1-G)^{1/2} e^{-i\beta} & iG^{1/2} \\ iG^{1/2} & (1-G)^{1/2} e^{i\beta} \end{vmatrix}$$

Here G is the dimension-less instantaneous conductance and is given by

$$G(t) = -\left| \frac{2\Gamma_R(t)\Gamma_L(t)}{\Gamma_R(t)^2 + \Gamma_L(t)^2 - iv\varepsilon} \right|^2 \qquad (5)$$

Without loss of generality, we have assumed that $\Gamma_{R/L}$ are real quantities in Eq. (5). Upon using the Brouwer's formula, the pumped charge can be be straight-forwardly obtained as

$$\mathcal{Q} = \frac{e^\star}{2\pi} \int_0^\tau dt \left[\frac{d\lambda(t)}{dt} - G(t) \frac{d\beta(t)}{dt} \right] \qquad (6)$$

where the integral is taken along the closed contour in the parameter space of $\Gamma_R - \Gamma_L$, during one pumping cycle and $\lambda = \theta - \beta$ is a reflection amplitude phase. The first term gives a quantized contribution to pumped charge in units of $e^\star = \nu e$. It is topological in nature and hence does not depend on the details of the contour. This is because $\lambda(\tau) = \lambda(0) + 2n\pi$ where n is an integer. The second term is the one which destroys quantization as it is directly proportional to the conductance.

From the above equation (Eq. (5)) we note that if the resonant level is tuned such that its position in energy space is infinitesimally close to the Fermi energy of the leads, then the line corresponding to $\Gamma_R = \Gamma_L$ in the $\Gamma_R - \Gamma_L$ plane corresponds to a line of perfect resonances *i.e.*, $G = 1$. To obtain quantized pumped charge it is essential that
(a) The topological contribution to pumped charge is non-zero. This is achieved by choosing the pumping contour to be such that, it encloses a finite portion of the line of resonances. This is because every time the pumping contour crosses the line of resonances, the phase of the reflection amplitude discontinuously changes by a factor of π. As the pumping contour will cross the line of resonances at least at two points for the case of simplest possible closed curve in the parameter plane, the total change in reflection phase θ over one complete pumping cycle amounts to a total change of 2π resulting in pumped charge of amount νe.
(b) It is also important to note that, in the expression for the pumped charge there is part which is proportional to conductance (dissipative part) which will destroy the quantization until and unless the pumping contour is such that the value of conductance $G(t)$ remains vanishingly small on the pumping contour. This can be achieved by tuning the couplings $\Gamma_{R/L}$ to be very small such that the resonance become sharp leading to vanishing of $G(t)$ in most part of the pumping contour, as is evident from Eq. (5). Now the only hurdle which still remains is related to the fact that the dissipative part picks up considerable contribution when the pumping contour cuts the line of resonances.
This hurdle is a consequence of the fact that we have used an over-simplified model for the resonant level. Generically, the position of the resonant level which corresponds to quasi-bound state in the anti-dot is not independent of $\Gamma_{R/L}$, but is expected to be a smooth function of $\Gamma_{R/L}$ [4]. Hence the line of perfect resonance at $\Gamma_R = \Gamma_L$ for a given Fermi energy in the lead will get restricted and will shrink to a set of points in the parameter space which can be enclosed completely within an appropriately chosen pumping contour leading to almost perfect quantization of pumped charge. We say "almost" because there will always be some small yet finite contribution coming from the tail of the resonance which will fall on the pumping contour. It is worth noticing that even when the pumping contour does not cross any line of resonance and only encloses a point of perfect resonance in the pumping parameter space, the phase of the reflection amplitude changes in multiples of 2π over a period τ, and is not a periodic function of time. This is because the resonance corresponds to zero refection ($r = 0$) and hence $\lambda = \mathcal{I}m\,[\log r]$ goes through a branch cut when the contour encloses the resonance which ultimately leads to the non-periodicity of reflection phase as a function of time.

Pumping for the case of $k \neq \nu$. – For the case of $k \neq 1/3$, we study the problem perturbatively in the parameter δk which represents a small deviation of k around the point, $k = 1/3$. Since this point corresponds to free fermionic theory, the case of $k = (1/3) \pm \delta k$ maps onto the problem of weakly interacting spin-less fermion (non-chiral Luttinger liquid) whose Luttinger parameter is given by $(k \pm \delta k)/\nu$ which is $1 \pm (\delta k/\nu)$ when $k = \nu$. The plus sign corresponds to the case of attractive fermions while the minus sign corresponds to repulsive fermions. Also we assume that $\delta k << \nu$. For the case of weakly interacting fermions in 1–D, it is possible to calculate transport through a localized scalar impurity [20] or a resonant

level [25] by calculating corrections to the free fermion scattering matrix element representing the quantum scatterer, perturbatively in the interaction strength followed by a "poor-man's scaling" approach which eventually gives an RG equation for the S-matrix element itself. As an alternative to bosonization, we adopt the above mentioned approach to first calculate the S-matrix elements at a given energy scale corresponding to the time-frozen Hamiltonian for the problem at hand. We then solve the RG equations for the S-matrix to obtain the energy scale dependence of these elements. Then using the Brouwer's formula, we calculate the pumped charge in the adiabatic limit for the case $k = \nu \pm \delta k$. The RG equation for transmission amplitude of the fermions through the resonant level, when the level is tuned infinitesimally close to the Fermi energy in the lead is given by [25]

$$\frac{dt^{\pm}_{R/L}}{d\ln(\Delta E/\tilde{\epsilon})} = \pm\left(\frac{\delta k}{\nu}\right)\left[t^{\pm}_{R/L}\left\{1-\left|t^{\pm}_{R/L}\right|^2\right\}\right] \quad (7)$$

$$\frac{dr^{\pm}_{R/L}}{d\ln(\Delta E/\tilde{\epsilon})} = \mp\left(\frac{\delta k}{\nu}\right)\Big[-r^{\pm}_{R/L}+r^{\pm}_{R/L}|r^{\pm}_{R/L}|^2 + t^{\pm}_{R/L}t^{\pm}_{L/R}r^{\pm}_{L/R}\Big] \quad (8)$$

Here, $t^{\pm}_{R/L}$ represents the transmission amplitude from the right (left) moving edge to the left (right) moving edge via the resonant level when the effective Luttinger liquid parameter is tuned to $1\pm\delta k/\nu$. Similarly, $r^{\pm}_{R/L}$ stands for the reflection amplitude in the right (left) moving edge respectively. And $\tilde{\epsilon}$ is the energy of the fermion measured form the Fermi-level. The ultra-violet cut-off scale in our case is set by the average level spacing of the levels in the anti-dot (ΔE). Now, given the fact that we are at zero bias and the pumping frequency is such that it is smaller then all the relevant energy scale ($\varepsilon, \Delta E$) in the problem to remain in the adiabatic limit, hence the only energy scale which acts as the low energy cutoff is energy scale set by temperature and we must stop the RG flow at energy corresponding to $|\epsilon| \simeq k_B T$. So, the solutions of the RG equation will provide us with the temperature dependence of the transmission and reflection amplitudes. Note that the small parameter corresponding to the interaction strength (say α) in terms of which the fermionic perturbation theory is developed is connected to the Luttinger parameter (say K) of the fermions via $K = \sqrt{(1-\alpha)/(1+\alpha)}$ [26]. As we are only interested in the weak interaction limit, *i.e.* the limit of $\alpha \to 0$ we expand K to first order in α to get $K = 1-\alpha$. For fermions with attractive interaction, we have $K > 1$ and α is negative while for the fermions with repulsive interaction, $K < 1$ and α is positive. From Eq. (7), it is easy to see that the transmission amplitude for the case of attractive fermions ($K = 1 + \delta k/\nu \Rightarrow \alpha = -\delta k/\nu$) grows under RG flow. Hence the RG flow will destroy the quasi-particle resonant level in the IR limit and quantization of pumped charge in units of νe cannot be achieved in this case. On the contrary, for the case of repulsive fermions ($K = 1 - \delta k/\nu \Rightarrow \alpha = \delta k/\nu$), Eq. (7) suggests that the transmission amplitude flows to zero under RG whenever the transmission through the level is different then unity when the IR limit is taken. Hence in this case the RG flow leads to an extremely sharp resonance resulting in quantization of pumped charge in units of νe in the IR limit. As a matter of fact, similar results regarding quantization of pumped charge were also obtained in Ref. [11, 27] for the case of $K < 1$ and $K > 1$.

Now, after obtaining the temperature dependence of the S-matrix elements for the level, we can obtain the temperature dependence of the pumped charge at a given temperature using Eq. (6). It is worth noticing that Eq. (7) leads to RG flow equation for both the transmission ($|t^{\pm}_{R/L}|$) and reflection amplitude ($|r^{\pm}_{R/L}|$) and the associated phases $\lambda = \theta - \beta, \theta$ respectively. Without loss of generality, we can choose to calculate the temperature dependence of the S-matrix element anywhere on the pumping contour in the space of pumping parameters (*i.e.* $\Gamma_R - \Gamma_L$). To avoid unnecessary complications arising due to the RG flow of phases associated with S-matrix elements (λ, θ), we choose to calculate the solution for the RG equation for the case of symmetric barrier. This symmetry leads to vanishing of the RG flow of the phase hence simplifying the calculation considerably. In doing so we have assumed that, the pumped charge at different temperature had been obtained by cooling the system when the coupling of the resonant level is tuned to be symmetric with respect to the left moving edge and the right moving edge *i.e.*, at those points on the pumping contour where $\Gamma_1 = \Gamma_2$ line cuts the pumping contour. The RG equation for the transmission amplitude for the symmetric case hence reduces to

$$\frac{d|t^{\pm}_{R/L}|}{dl} = \pm\left(\frac{\delta k}{\nu}\right)\left[\;|t^{\pm}_{R/L}|\left\{1-|t^{\pm}_{R/L}|^2\right\}\;\right] \quad (9)$$

We now integrate the RG equation for obtaining the power-law dependence for the dimension-less conductance$((|t^{\pm}_{R/L}|^2))$ as a function of temperature as

$$G^{\pm}(T) = T_0^{\pm}\frac{\left(\frac{\Delta E}{k_B T}\right)^{\pm 2(\delta k/\nu)}}{\left[R_0^{\pm}+T_0^{\pm}\left\{\left(\frac{\Delta E}{k_B T}\right)^{\pm 2(\delta k/\nu)}\right\}\right]} \quad (10)$$

Here $T_0^{\pm}$ and $R_0^{\pm}$ are the values of $T^{\pm}$ and $R^{\pm}$ at $L_T = d$. Hence using Eqs. (6) and (10), we obtain the pumped charge as a function of temperature which is given by

$$\mathcal{Q}^{\pm} = \mathcal{Q}_{\rm int} - \left(\frac{\Delta E}{k_B T}\right)^{\pm 2(\delta k/\nu)}\int_0^{\tau} dt\; I^{\pm}(t) \quad (11)$$

where

$$I^{\pm}(t) = \frac{e^{\star}}{2\pi}\frac{G_0^{\pm}\left[\dot{\beta}\right]}{1+G_0^{\pm}\left[-1+\left(\frac{\Delta E}{k_B T}\right)^{\pm 2\delta k/\nu}\right]}$$

Here $I^{\pm}(t)$ represents the instantaneous dissipative current which spoils the quantization of pumped charge. From Eq. (11), it is easy to see that for the case of $k > 1/3$ *i.e.*, for $\mathcal{Q}^{+}$, quantization of the pumped charge becomes worse as we go to lower temperatures, which is consistent with the observation that for $k = 1$ the quantization is in integer units of e, whereas for the case of $k < 1/3$ *i.e.*, for $\mathcal{Q}^{-}$, the quantization of the pumped charge is achieved as we go to lower temperatures limit (IR limit).

Conclusions and Discussion. – In conclusion, in this letter we obtain an exact expression for the pumped charge in the adiabatic limit for a specific strength of the inter-edge interaction, namely for $k = 1/3$. We also obtain an expression for pumped charge for any small deviation around the value of $k = 1/3$ perturbatively in the small parameter which is taken to be the deviation of k from its value of one-thirds and show that for $k \leq 1/3$ the charge pumped in a cycle is quantized in units of νe. We also propose a experimental set-up in which our prediction regarding quantization of pumped charge in units of νe can be verified. In the end, it is important to emphasize that for temperatures $T < T_{L/2}$ ($T_{L/2}$ being the temperature corresponding to the distance $L/2$, the length of each line junction), there will be correlation to the power law dependance of the conductance due to junction between the edge of the line junction and the freely propagating edge connected to the contacts (Fig. 1(b)). This would imply that the results quoted in Eq. (11) should get appropriately modified for temperatures $T < T_{L/2}$. To avoid this complication one can fabricate ohmic contacts right at the end of the two line junctions rather than having it far from the line junction as in Fig. 1(b) and hence for $T < T_{L/2}$, the temperature power laws in Eq. (11) are to be replaced by length power laws.

Acknowledgements. – We thank Yuval Gefen for many stimulating discussions on the physics of edge state, comments on the manuscript and encouragement. SD gratefully acknowledges Sumathi Rao, Efrat Shimshoni and Izhar Neder for useful discussions. We acknowledge financial support from the Feinberg Fellowship at Weizmann Institute of Science, Israel, the US−Israel BSF, the Albert Einstein Minerva Center for Theoretical Physics and a grant from the Minerva Foundation.

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